Design and analysis of Walsh-Tukey function systems of any order

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Walsh systems are useful in various applications across different fields of science and engineering. They are especially useful in signal processing where signals are represented as a series of discrete values. Due to orthogonal properties, it helps in analysis and transforming digital signals [1]. Walsh functions are also applicable in cryptographic algorithms due to their orthogonal and pseudo-random properties. Also, it has applications in: CDMA (Code Division Multiple Access) systems, image processing, digital data filtering.

A complete system of Walsh functions creates a Walsh's space basis, where each function is called a basis function of the N order [2]. All basis functions are represented as a symmetric matrix of N order. We can name three classic systems: Walsh-Hadamard, Walsh- Kaczmage, Walsh-Paley, but none of the three classic bases provide linear connectivity to the frequency scales of DFT processors, which makes their usage in spectral analysis more complicated. Meanwhile Walsh-Tukey bases provide needed linear connectivity, that comes from frequency scales ratio shown on Figure 1.

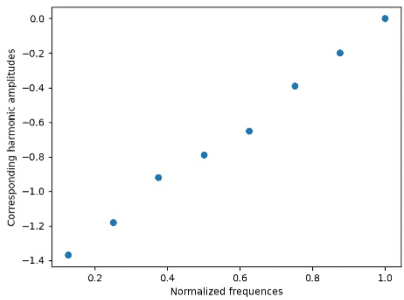

Fig.1. Frequency scales ratio of DFT processors in Walsh-Tukey basis

Designing algorithm of the all Walsh-Tukey basis functions shown on Figure 2. Result of the algorithm is the N-order matrix. Firstly, we fill-in initial zero and first rows. Where zero rows fully consist of the 0 (+1) values and the first row contains half of the N non-zero values 1 (-1), other places are filled by zero elements. Each row in the matrix equals the column with the same index. By next three steps we do: filling even rows, filling odd rows, rows supplementation. Even rows are filled using expression (1).

$$(2k, t) = (k, 2t) \tag{1}$$

Which means that the first half of the even row copies even elements of the row whose index is two times less than current. Outstanding elements can be easily added following the rule that the second half of even rows copy the first one.

Odd rows can be filled using expression (2). Where the odd row is fully copied from the selected even row.

$$row_{odd} = (N + 1) - row_{even} \quad (2)$$

And the final third step is a search over rows where elements of one half of the row are copying elements from another for even rows and for odd rows elements of the one half are inverted to another.

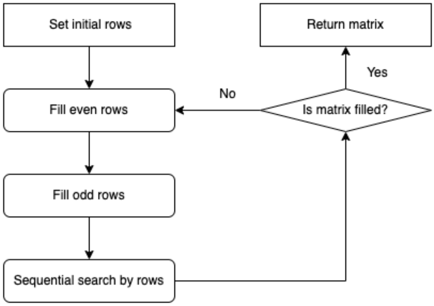

Fig.2. Block diagram of the Walsh-Tukey system designing algorithm

If the matrix is not filled after the third step, the algorithm should be repeated from the first step. Algorithm complexity is $O(log\ N)$.

The article details an efficient algorithm for constructing Walsh-Tukey basis functions and its theoretical insights and practical guidelines for implementing Walsh-Tukey systems in technological applications, emphasizing their significance in advancing digital communication and processing technologies.

1. J. Walsh, "A closed set of normal orthogonal functions", Amer. J. Math., v. 45, pp. 5-24, 1923.
2. БелецкийА. Я. Оптимальные Уолша и Уолше-подобные базисы дискретного преобразования Фурье. / А. Я. Белецкий. // Захист інформації. — 2018. – Т. 20, № 2. – С. 104-119.